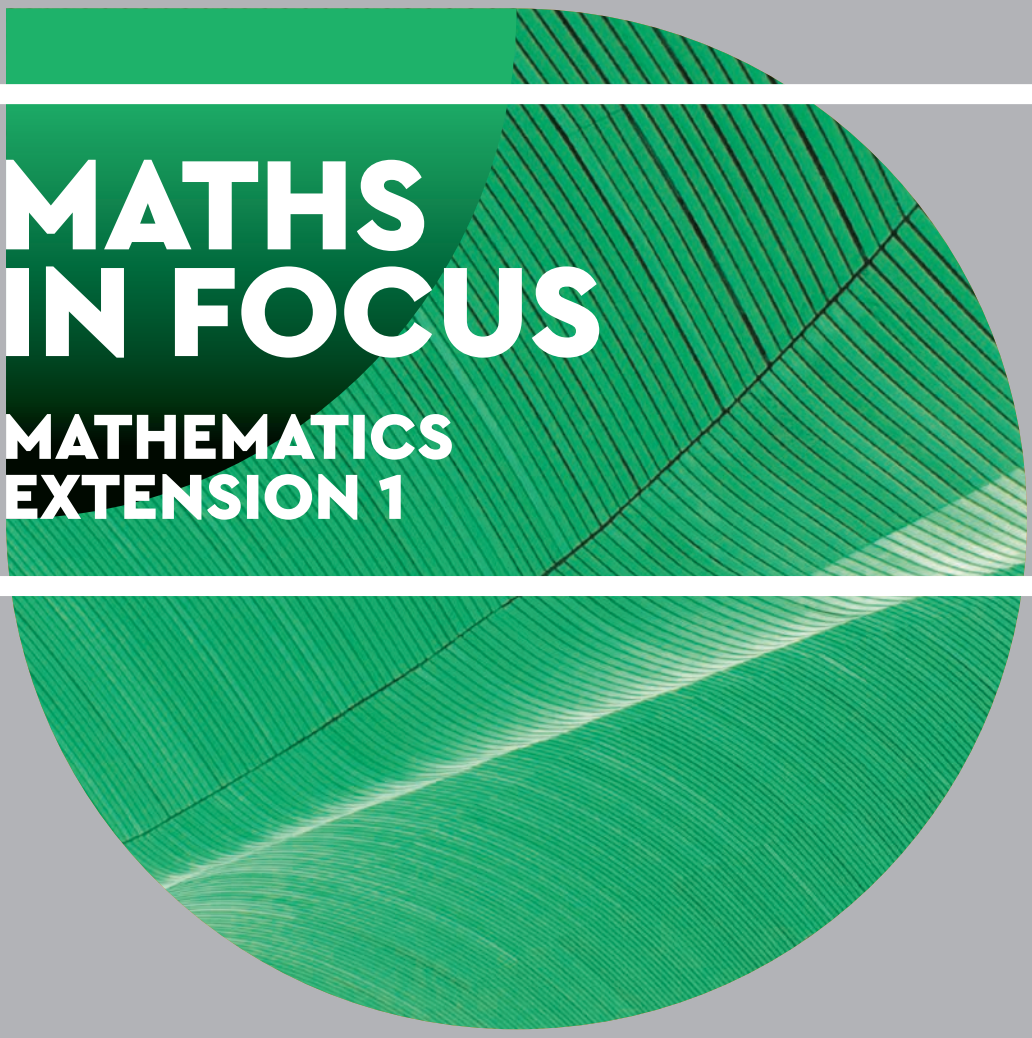

MATHS IN FOCUS

MATHEMATICS EXTENSION 1

YEAR 11

Margaret Grove
Series editor: Robert Yen

3RD EDITION

NELSON
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PREFACE

Maths in Focus 11 Mathematics Extension 1 has been rewritten for the new Mathematics Extension 1 syllabus (2017). In this 3rd edition of the book, teachers will find those familiar features that have made Maths in Focus a leading senior mathematics series, such as clear and abundant worked examples in plain English, comprehensive sets of graded exercises, chapter *Test Yourself* and *Challenge* exercises, Investigations, and practice sets of mixed revision and exam-style questions.

The Mathematics Extension 1 course is designed for students who intend to study mathematics at university, possibly majoring in the subject.

This book covers the Year 11 content of the course, which includes the Year 11 Mathematics Advanced course. The specific Mathematics Extension 1 content is labelled EXT1. The theory follows a logical order, although some topics may be learned in any order. We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant and meaningful applications of mathematics.

The *NelsonNet* student and teacher websites contain additional resources such as worksheets, video tutorials and topic tests. We wish all teachers and students using this book every success in embracing the new senior mathematics course.

ABOUT THE AUTHOR

Margaret Grove has spent over 30 years teaching HSC Mathematics, most recently at Bankstown TAFE College. She has written numerous senior mathematics texts and study guides over the past 25 years, including the bestselling *Maths in Focus* series for Mathematics and Mathematics Extension 1.

Margaret thanks her family, especially her husband Geoff, for their support in writing this book.

CONTRIBUTING AUTHORS

Gaspare Carrozza and **Haroon Ha** from Homebush Boys High School wrote many of the *NelsonNet* worksheets.

Scott Smith and **Cherylanne Saywell** created the video tutorials.

Tania Eastcott and **Elizabeth Nabhan** wrote the topic tests.

Roger Walter wrote the *ExamView* questions.

Shane Scott, **Brandon Pettis** and **George Dimitriadis** wrote the worked solutions to all exercise sets.

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* = Revision

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F1.3 Linear, quadratic and cubic functions	4 Functions
F1.4 Further functions and relations	7 Further functions
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Topic and subtopic	*Maths in Focus 11* *Mathematics Extension 1 chapter*
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MA-S1 Probability and discrete probability distributions	
S1.1 Probability and Venn diagrams S1.2 Discrete probability distributions	9 Probability 12 Discrete probability distributions
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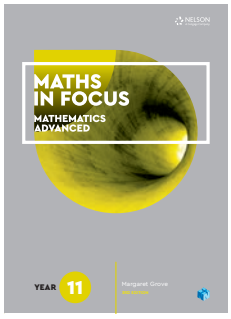
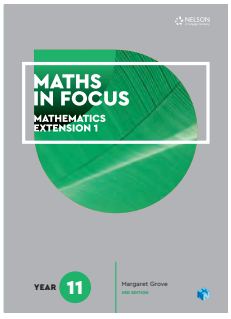
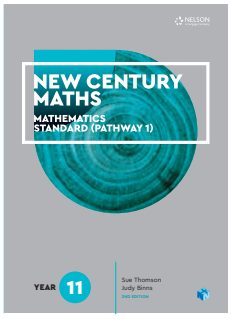
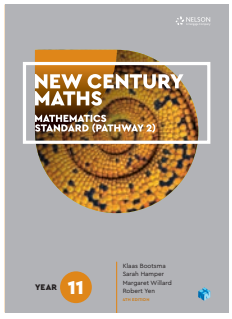
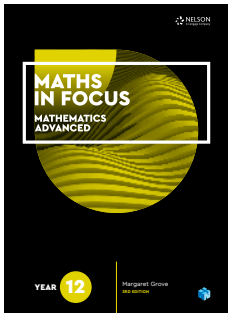
MATHS IN FOCUS AND NEW CENTURY MATHS 11–12

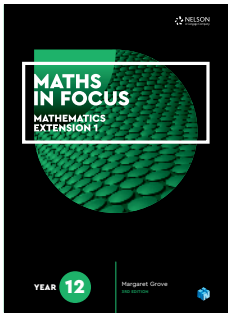


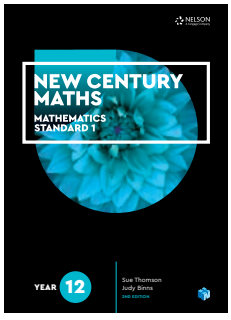


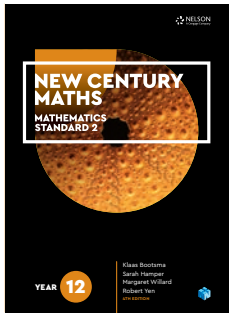


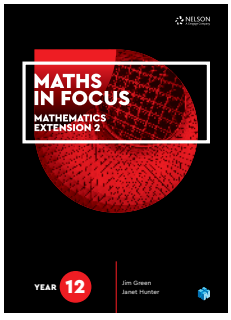












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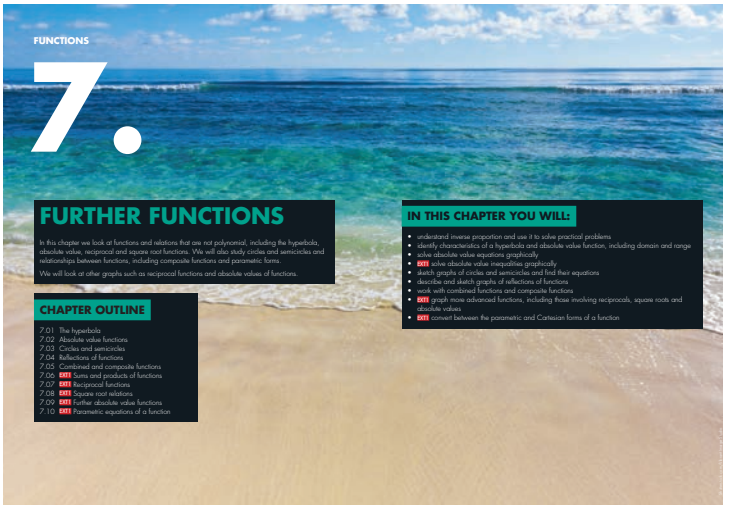
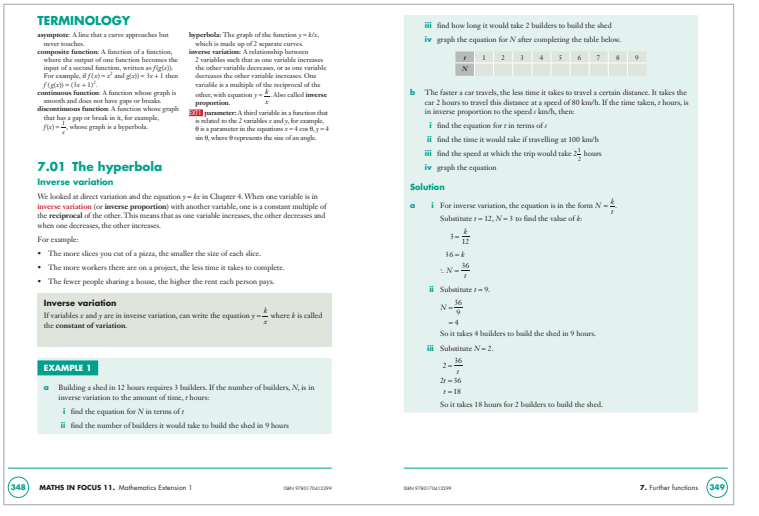
ABOUT THIS BOOK

AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread showing the **Chapter contents** and a list of chapter outcomes



- **Terminology** is a chapter glossary that previews the key words and phrases from within the chapter



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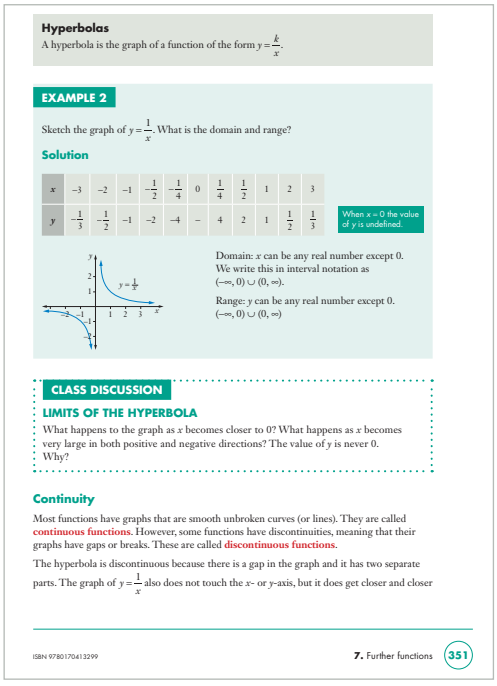
IN EACH CHAPTER

- Important facts and formulas are highlighted in a shaded box.
- Important words and phrases are printed in red and listed in the Terminology chapter glossary.
- The specific Mathematics Extension 1 content is labelled EXT1.
- Graded exercises include exam-style problems and realistic applications.
- Worked solutions to all exercise questions are provided on the *NelsonNet* teacher website.
- **Investigations** explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks.
- **Did you know?** contains interesting facts and applications of the mathematics learned in the chapter.



Hyperbolas

A hyperbola is the graph of a function of the form $y = \frac{k}{x}$.

EXAMPLE 2

Sketch the graph of $y = \frac{1}{x}$. What is the domain and range?

Solution

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-4	–	4	2	1	$\frac{1}{2}$	$\frac{1}{3}$

When $x = 0$ the value of y is undefined.

Domain: x can be any real number except 0.
We write this in interval notation as $(-\infty, 0) \cup (0, \infty)$.

Range: y can be any real number except 0.
$(-\infty, 0) \cup (0, \infty)$

CLASS DISCUSSION

LIMITS OF THE HYPERBOLA

What happens to the graph as x becomes closer to 0? What happens as x becomes very large in both positive and negative directions? The value of y is never 0. Why?

Continuity

Most functions have graphs that are smooth unbroken curves (or lines). They are called **continuous functions**. However, some functions have discontinuities, meaning that their graphs have gaps or breaks. These are called **discontinuous functions**.

The hyperbola is discontinuous because there is a gap in the graph and it has two separate parts. The graph of $y = \frac{1}{x}$ also does not touch the x- or y-axis, but it does get closer and closer

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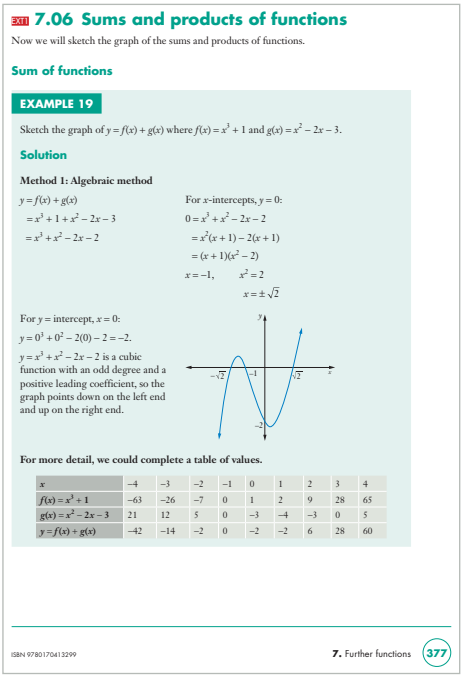

EXT1 7.06 Sums and products of functions

Now we will sketch the graph of the sums and products of functions.

Sum of functions

EXAMPLE 19

Sketch the graph of $y = f(x) + g(x)$ where $f(x) = x^3 + 1$ and $g(x) = x^2 - 2x - 3$.

Solution

Method 1: Algebraic method

$y = f(x) + g(x)$
$= x^3 + 1 + x^2 - 2x - 3$
$= x^3 + x^2 - 2x - 2$

For x-intercepts, $y = 0$:
$0 = x^3 + x^2 - 2x - 2$
$= x^2(x + 1) - 2(x + 1)$
$= (x + 1)(x^2 - 2)$
$x = -1, \quad x^2 = 2$
$x = \pm\sqrt{2}$

For y = intercept, $x = 0$:
$y = 0^3 + 0^2 - 2(0) - 2 = -2$.

$y = x^3 + x^2 - 2x - 2$ is a cubic function with an odd degree and a positive leading coefficient, so the graph points down on the left end and up on the right end.

For more detail, we could complete a table of values.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = x^3 + 1$	-63	-26	-7	0	1	2	9	28	65
$g(x) = x^2 - 2x - 3$	21	12	5	0	-3	-4	-3	0	5
$y = f(x) + g(x)$	-42	-14	-2	0	-2	-2	6	28	60

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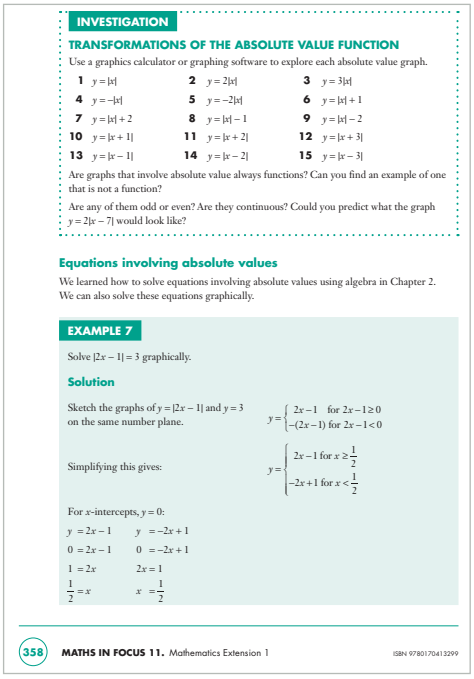

INVESTIGATION

TRANSFORMATIONS OF THE ABSOLUTE VALUE FUNCTION

Use a graphics calculator or graphing software to explore each absolute value graph.

1 $y = \lvert x \rvert$	**2** $y = 2\lvert x \rvert$	**3** $y = 3\lvert x \rvert$
4 $y = -\lvert x \rvert$	**5** $y = -2\lvert x \rvert$	**6** $y = \lvert x \rvert + 1$
7 $y = \lvert x \rvert + 2$	**8** $y = \lvert x \rvert - 1$	**9** $y = \lvert x \rvert - 2$
10 $y = \lvert x + 1 \rvert$	**11** $y = \lvert x + 2 \rvert$	**12** $y = \lvert x + 3 \rvert$
13 $y = \lvert x - 1 \rvert$	**14** $y = \lvert x - 2 \rvert$	**15** $y = \lvert x - 3 \rvert$

Are graphs that involve absolute value always functions? Can you find an example of one that is not a function?

Are any of them odd or even? Are they continuous? Could you predict what the graph $y = 2|x - 7|$ would look like?

Equations involving absolute values

We learned how to solve equations involving absolute values using algebra in Chapter 2. We can also solve these equations graphically.

EXAMPLE 7

Solve $|2x - 1| = 3$ graphically.

Solution

Sketch the graphs of $y = |2x - 1|$ and $y = 3$ on the same number plane.

$$y = \begin{cases} 2x - 1 & \text{for } 2x - 1 \geq 0 \\ -(2x - 1) & \text{for } 2x - 1 < 0 \end{cases}$$

Simplifying this gives:

$$y = \begin{cases} 2x - 1 \text{ for } x \geq \frac{1}{2} \\ -2x + 1 \text{ for } x < \frac{1}{2} \end{cases}$$

For x-intercepts, $y = 0$:

$y = 2x - 1$ $\quad$ $y = -2x + 1$
$0 = 2x - 1$ $\quad$ $0 = -2x + 1$
$1 = 2x$ $\quad$ $2x = 1$
$\frac{1}{2} = x$ $\quad$ $x = \frac{1}{2}$

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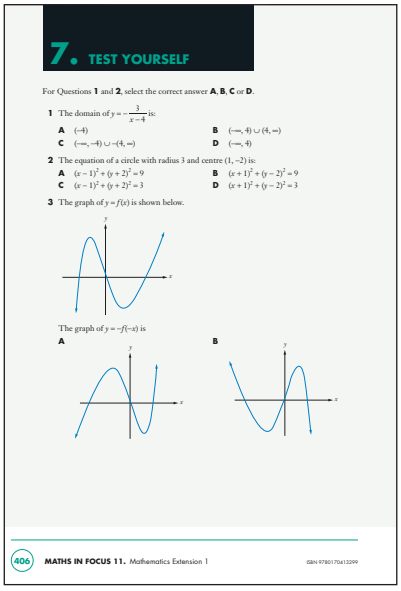
AT THE END OF EACH CHAPTER

- **Test Yourself** contains chapter revision exercises.
- If you have trouble completing the *Test Yourself* exercises, you need to go back and revise the chapter before trying the exercises again.
- **Challenge Exercise** contains chapter extension questions. Attempt these only after you are confident with the *Test Yourself* exercises, because these are more difficult and are designed for students who understand the topic really well.
- **Practice sets** (after several chapters) provide a comprehensive variety of mixed exam-style questions from various chapters, including short-answer, free-response and multiple-choice questions.


7. TEST YOURSELF

For Questions 1 and 2, select the correct answer A, B, C or D.

1 The domain of $y = \frac{3}{x+4}$ is:

A (-4) B $(-\infty, 4) \cup (4, \infty)$

C $(-\infty, -4) \cup (-4, \infty)$ D $(-\infty, 4)$

2 The equation of a circle with radius 3 and centre $(1, -2)$ is:

A $(x-1)^2 + (y+2)^2 = 9$ B $(x+1)^2 + (y-2)^2 = 9$

C $(x-1)^2 + (y+2)^2 = 3$ D $(x+1)^2 + (y-2)^2 = 3$

3 The graph of $y = f(x)$ is shown below.

The graph of $y = -f(-x)$ is

A B

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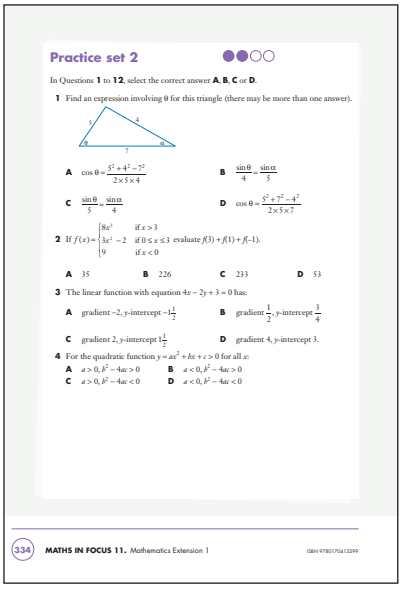
AT THE END OF THE BOOK

- **Answers** and **Index** (worked solutions on the teacher website).


Practice set 2

In Questions 1 to 12, select the correct answer A, B, C or D.

1 Find an expression involving θ for this triangle (there may be more than one answer).

A $\cos\theta = \frac{5^2 + 4^2 - 7^2}{2 \times 5 \times 4}$ B $\frac{\sin\theta}{4} = \frac{\sin\alpha}{5}$

C $\frac{\sin\theta}{5} = \frac{\sin\alpha}{4}$ D $\cos\theta = \frac{5^2 + 7^2 - 4^2}{2 \times 5 \times 7}$

2 If $f(x) = \begin{cases} 8x^3 & \text{if } x > 3 \\ 3x^2 - 2 & \text{if } 0 \le x \le 3 \\ 9 & \text{if } x < 0 \end{cases}$ evaluate $f(3) + f(1) + f(-1)$.

A 35 B 226 C 233 D 53

3 The linear function with equation $4x - 2y + 3 = 0$ has:

A gradient -2, y-intercept $-1\frac{1}{2}$ B gradient $\frac{1}{2}$, y-intercept $\frac{3}{4}$

C gradient 2, y-intercept $1\frac{1}{2}$ D gradient 4, y-intercept 3.

4 For the quadratic function $y = ax^2 + bx + c > 0$ for all x:

A $a > 0, b^2 - 4ac > 0$ B $a < 0, b^2 - 4ac > 0$

C $a > 0, b^2 - 4ac < 0$ D $a < 0, b^2 - 4ac < 0$

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NELSONNET STUDENT WEBSITE

Margin icons link to print (PDF) and multimedia resources found on the *NelsonNet* student website, **www.nelsonnet.com.au**. These include:

- **Worksheets** and **puzzle sheets** that are write-in enabled PDFs
- **Video tutorials:** worked examples explained by 'flipped classroom' teachers
- ***ExamView* quizzes:** interactive and self-marking

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NELSONNET TEACHER WEBSITE

The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

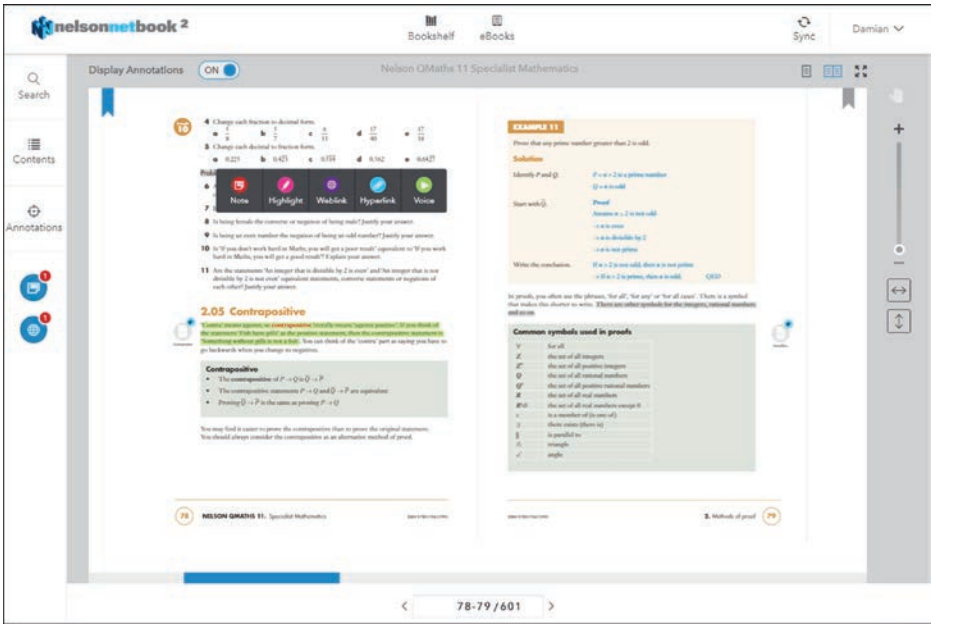
- A **teaching program,** in Microsoft Word and PDF formats
- **Topic tests,** in Microsoft Word and PDF formats
- **Worked solutions** to each exercise set
- **Chapter PDFs** of the textbook
- ***ExamView*** exam-writing software and questionbanks
- **Resource Finder:** search engine for *NelsonNet* resources

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

NELSONNETBOOK

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students



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STUDY SKILLS

The Year 11 course introduces the basics of topics such as calculus that are then applied in the Year 12 course. You will struggle in the HSC if you don't set yourself up to revise the Year 11 topics as you learn new Year 12 topics. Your teachers will be able to help you build up and manage good study habits. Here are a few hints to get you started. There is no right or wrong way to learn. Different styles of learning suit different people. There is also no magical number of hours a week that you should study, because this will be different for every student. But just listening in class and taking notes is not enough, especially when you are learning material that is totally new.

If a skill is not practised within the first 24 hours, up to 50% can be forgotten. If it is not practised within 72 hours, up to 85–90% can be forgotten! So it is really important that, whatever your study timetable, new work must be looked at soon after it is presented to you.

With a continual succession of new work to learn and retain, this is a challenge. But the good news is that you don't have to study for hours on end!

IN THE CLASSROOM

In order to remember, first you need to focus on what is being said and done.

According to an ancient proverb:

I hear and I forget
I see and I remember
I do and I understand.

If you chat to friends and just take notes without really paying attention, you aren't giving yourself a chance to remember anything and will have to study harder at home.

If you are unsure of something that the teacher has said, the chances are that others are also not sure. Asking questions and clarifying things will ultimately help you gain better results, especially in a subject like mathematics where much of the knowledge and skills depend on being able to understand the basics.

Learning is all about knowing what you know and what you don't know. Many students feel like they don't know anything, but it's surprising just how much they know already. Picking up the main concepts in class and not worrying too much about other less important parts can really help. The teacher can guide you on this.

Here are some pointers to get the best out of classroom learning:

- Take control and be responsible for your own learning
- Clear your head of other issues in the classroom
- Active, not passive, learning is more memorable
- Ask questions if you don't understand something

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- Listen for cues from the teacher
- Look out for what are the main concepts.

Note-taking varies from class to class, but here are some general guidelines:

- Write legibly
- Use different colours to highlight important points or formulas
- Make notes in textbooks (using pencil if you don't own the textbook)
- Use highlighter pens to point out important points
- Summarise the main points
- If notes are scribbled, rewrite them at home.

AT HOME

You are responsible for your own learning and nobody else can tell you how best to study. Some people need more revision time than others, some study better in the mornings while others do better at night, and some can work at home while others prefer a library.

- Revise both new and older topics regularly
- Have a realistic timetable and be flexible
- Summarise the main points
- Revise when you are fresh and energetic
- Divide study time into smaller rather than longer chunks
- Study in a quiet environment
- Have a balanced life and don't forget to have fun!

If you are given exercises out of a textbook to do for homework, consider asking the teacher if you can leave some of them till later and use these for revision. It is not necessary to do every exercise at one sitting, and you learn better if you can spread these over time.

People use different learning styles to help them study. The more variety the better, and you will find some that help you more than others. Some people (around 35%) learn best visually, some (25%) learn best by hearing and others (40%) learn by doing.

- Summarise on cue cards or in a small notebook
- Use colourful posters
- Use mindmaps and diagrams
- Discuss work with a group of friends
- Read notes out aloud
- Make up songs and rhymes
- Exercise regularly
- Role-play teaching someone else

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ASSESSMENT TASKS AND EXAMS

You will cope better in exams if you have practised doing sample exams under exam conditions. Regular revision will give you confidence, and if you feel well prepared this will help get rid of nerves in the exam. You will also cope better if you have had a reasonable night's sleep before the exam.

One of the biggest problems students have with exams is in timing. Make sure you don't spend too much time on questions you're unsure about, but work through and find questions you can do first.

Divide the time up into smaller chunks for each question and allow some extra time to go back to questions you couldn't do or finish. For example, in a 2-hour exam with 6 questions, allow around 15 minutes for each question. This will give an extra half hour at the end to tidy up and finish off questions.

- Read through and ensure you know how many questions there are
- Divide your time between questions with extra time at the end
- Don't spend too much time on one question
- Read each question carefully, underlining key words
- Show all working out, including diagrams and formulas
- Cross out mistakes with a single line so it can still be read
- Write legibly

AND FINALLY...

Study involves knowing what you don't know, and putting in a lot of time into concentrating on these areas. This is a positive way to learn. Rather than just saying, 'I can't do this', say instead, 'I can't do this yet', and use your teachers, friends, textbooks and other ways of finding out.

With the parts of the course that you do know, make sure you can remember these easily under exam pressure by putting in lots of practice.

Remember to look at new work:

today, tomorrow, in a week, in a month.

Some people hardly ever find time to study while others give up their outside lives to devote their time to study. The ideal situation is to balance study with other aspects of your life, including going out with friends, working, and keeping up with sport and other activities that you enjoy.

Good luck with your studies!

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MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation

apply: use knowledge or a procedure in a given situation

classify, identify: state the type, name or feature of an item or situation

comment: express an observation or opinion about a result

compare: show how two or more things are similar or different

construct: draw an accurate diagram

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately

evaluate, calculate: find the value of a numerical expression, for example 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression, for example expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

factorise: opposite to **expand,** to insert brackets by taking out a common factor, for example factorising $6y + 3$ gives $3(2y + 1)$

give reasons: show the rules or thinking used when solving a problem. *See also* **justify.**

hence find/prove: find an answer or prove a result using previous answers or information supplied

interpret: find meaning in a mathematical result

justify: give reasons or evidence to support your argument or conclusion. *See also* **give reasons**

rationalise: make rational, remove surds

show that, prove: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

simplify: give a result in its most basic, shortest, neatest form, for example simplifying a ratio or algebraic expression

sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**

solve: find the value(s) of an unknown pronumeral in an equation or inequality

substitute: replace a variable by a number and evaluate

verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write, state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

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FUNCTIONS

1.

ALGEBRAIC TECHNIQUES

This chapter revises and extends the algebraic techniques that you will need for this course. These include indices, algebraic expressions, expansion, factorisation, algebraic fractions and surds.

CHAPTER OUTLINE

IN THIS CHAPTER YOU WILL:

- identify and use index rules including fractional and negative indices
- simplify algebraic expressions
- remove grouping symbols including perfect squares and the difference of 2 squares
- factorise expressions including binomials and special factors
- simplify algebraic fractions
- use algebra to substitute into formulas
- simplify and use surds including rationalising the denominator

TERMINOLOGY

binomial: A mathematical expression consisting of 2 terms; for example, $x + 3$ and $3x - 1$

binomial product: The product of binomial expressions; for example, $(x + 3)(2x - 1)$

expression: A mathematical statement involving numbers, pronumerals and symbols; for example, $2x - 3$

factor: A whole number that divides exactly into another number. For example, 4 is a factor of 28

factorise: To write an expression as a product of its factors; that is, take out the highest common factor in an expression and place the rest in brackets. For example, $2y - 8 = 2(y - 4)$

index: The power or exponent of a number. For example, 2^3 has a base number of 2 and an index of 3. The plural of index is **indices**

power: The index or exponent of a number. For example, 2^3 has a base number of 2 and a power of 3

root: A number that when multiplied by itself a given number of times equals another number. For example, $\sqrt{25} = 5$ because $5^2 = 25$

surd: A root that can't be simplified; for example, $\sqrt{3}$

term: A part of an expression containing pronumerals and/or numbers separated by an operation such as +, −, × or ÷. For example, in $2x - 3$ the terms are $2x$ and 3

trinomial: An expression with 3 terms; for example, $3x^2 - 2x + 1$

1.01 Index laws

An **index** (or **power** or **exponent**) of a number shows how many times a number is multiplied by itself. A **root** of a number is the inverse of the power.

For example:

- $4^3 = 4 \times 4 \times 4 = 64$
- $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
- $\sqrt{36} = 6$ since $6^2 = 36$
- $\sqrt[3]{8} = 2$ since $2^3 = 8$
- $\sqrt[6]{64} = 2$ since $2^6 = 64$

Note: In 4^3 the 4 is called the base number and the 3 is called the index or power.

There are some general laws that simplify calculations with indices. These laws work for any m and n, including fractions and negative numbers.

Index laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

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EXAMPLE 1

Simplify:

a $m^9 \times m^7 \div m^2$ **b** $(2y^4)^3$ **c** $\dfrac{(y^6)^3 \times y^{-4}}{y^5}$

Solution

a $m^9 \times m^7 \div m^2 = m^{9+7-2}$
$= m^{14}$

b $(2y^4)^3 = 2^3(y^4)^3$
$= 2^3 y^{4\times 3}$
$= 8y^{12}$

c $\dfrac{(y^6)^3 \times y^{-4}}{y^5} = \dfrac{y^{18} \times y^{-4}}{y^5}$
$= \dfrac{y^{18+(-4)}}{y^5}$
$= \dfrac{y^{14}}{y^5}$
$= y^{14-5}$
$= y^9$

Exercise 1.01 Index laws

1 Evaluate without using a calculator:

a $5^3 \times 2^2$ **b** $3^4 + 8^2$ **c** $\left(\dfrac{1}{4}\right)^3$

d $\sqrt[3]{27}$ **e** $\sqrt[4]{16}$

2 Evaluate correct to 1 decimal place:

a 3.7^2 **b** $1.06^{1.5}$ **c** $2.3^{-0.2}$

d $\sqrt[3]{19}$ **e** $\sqrt[3]{34.8 - 1.2 \times 43.1}$ **f** $\dfrac{1}{\sqrt[3]{0.99 + 5.61}}$

3 Simplify:

a $a^6 \times a^9 \times a^2$ **b** $y^3 \times y^{-8} \times y^5$ **c** $a^{-1} \times a^{-3}$

d $w^{\frac{1}{2}} \times w^{\frac{1}{2}}$ **e** $x^6 \div x$ **f** $p^3 \div p^{-7}$

g $\dfrac{y^{11}}{y^5}$ **h** $(x^7)^3$ **i** $(2x^5)^2$

j $(3y^{-2})^4$ **k** $a^3 \times a^5 \div a^7$ **l** $\left(\dfrac{x^2}{y^9}\right)^5$

m $\dfrac{w^6 \times w^7}{w^3}$ **n** $\dfrac{p^2 \times (p^3)^4}{p^9}$ **o** $\dfrac{x^6 \div x^7}{x^2}$

p $\dfrac{a^2 \times (b^2)^6}{a^4 \times b^9}$ **q** $\dfrac{(x^2)^{-3} \times (y^3)^2}{x^{-1} \times y^4}$

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4 Simplify:

a $x^5 \times x^9$ **b** $a^{-1} \times a^{-6}$ **c** $\dfrac{m^7}{m^3}$

d $k^{13} \times k^6 \div k^9$ **e** $a^{-5} \times a^4 \times a^{-7}$ **f** $x^{\frac{2}{5}} \times x^{\frac{3}{5}}$

g $\dfrac{m^5 \times n^4}{m^4 \times n^2}$ **h** $\dfrac{p^{\frac{1}{2}} \times p^{\frac{1}{2}}}{p^2}$ **i** $(3x^{11})^2$

j $\dfrac{(x^4)^6}{x^3}$

5 Expand each expression and simplify where possible:

a $(pq^3)^5$ **b** $\left(\dfrac{a}{b}\right)^8$ **c** $\left(\dfrac{4a}{b^4}\right)^3$

d $(7a^5b)^2$ **e** $\dfrac{(2m^7)^3}{m^4}$ **f** $\dfrac{xy^3 \times (xy^2)^4}{xy}$

g $\dfrac{(2k^8)^4}{(6k^3)^3}$ **h** $(2y^5)^7 \times \dfrac{y^{12}}{8}$ **i** $\left(\dfrac{a^6 \times a^4}{a^{11}}\right)^{-3}$

j $\left(\dfrac{5xy^9}{x^8 \times y^3}\right)^3$

6 Evaluate a^3b^2 when $a = 2$ and $b = \dfrac{3}{4}$.

7 If $x = \dfrac{2}{3}$ and $y = \dfrac{1}{9}$, find the value of $\dfrac{x^3y^2}{xy^5}$.

8 If $a = \dfrac{1}{2}$, $b = \dfrac{1}{3}$ and $c = \dfrac{1}{4}$, evaluate $\dfrac{a^2b^3}{c^4}$ as a fraction.

9 a Simplify $\dfrac{a^{11}b^8}{a^8b^7}$.

b Hence evaluate $\dfrac{a^{11}b^8}{a^8b^7}$ as a fraction when $a = \dfrac{2}{5}$ and $b = \dfrac{5}{8}$.

10 a Simplify $\dfrac{p^5q^8r^4}{p^4q^6r^2}$.

b Hence evaluate $\dfrac{p^5q^8r^4}{p^4q^6r^2}$ as a fraction when $p = \dfrac{7}{8}$, $q = \dfrac{2}{3}$ and $r = \dfrac{3}{4}$.

11 Evaluate $(a^4)^3$ when $a = \left(\dfrac{2}{3}\right)^{\frac{1}{6}}$.

12 Evaluate $\dfrac{a^3b^6}{b^4}$ when $a = \dfrac{1}{2}$ and $b = \dfrac{2}{3}$.

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13 Evaluate $\frac{x^4 y^7}{x^5 y^5}$ when $x = \frac{1}{3}$ and $y = \frac{2}{9}$.

14 Evaluate $\frac{k^{-5}}{k^{-9}}$ when $k = \frac{1}{3}$.

15 Evaluate $\frac{a^4 b^6}{a^3 (b^2)^2}$ when $a = \frac{3}{4}$ and $b = \frac{1}{9}$.

16 Evaluate $\frac{a^6 \times b^3}{a^5 \times b^2}$ as a fraction when $a = \frac{1}{9}$ and $b = \frac{3}{4}$.

1.02 Zero and negative indices

Zero and negative indices

$$x^0 = 1$$
$$x^{-n} = \frac{1}{x^n}$$

WS Review of index laws

EXAMPLE 2

a Simplify $\left(\frac{ab^5c}{abc^4}\right)^0$.

b Evaluate 2^{-3}.

c Write in index form:

i $\frac{1}{x^2}$ **ii** $\frac{3}{x^5}$ **iii** $\frac{1}{5x}$ **iv** $\frac{1}{x+1}$

d Write a^{-3} without the negative index.

Solution

a $\left(\frac{ab^5c}{abc^4}\right)^0 = 1$

b $2^{-3} = \frac{1}{2^3}$
$= \frac{1}{8}$

c **i** $\frac{1}{x^2} = x^{-2}$

ii $\frac{3}{x^5} = 3 \times \frac{1}{x^5}$
$= 3x^{-5}$

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iii $\frac{1}{5x} = \frac{1}{5} \times \frac{1}{x}$

$= \frac{1}{5}x^{-1}$

iv $\frac{1}{x+1} = \frac{1}{(x+1)^1}$

$= (x+1)^{-1}$

d $a^{-3} = \frac{1}{a^3}$

Exercise 1.02 Zero and negative indices

1 Evaluate as a fraction or whole number:

a 3^{-3} **b** 4^{-1} **c** 7^{-3} **d** 10^{-4} **e** 2^{-8}

f 6^{0} **g** 2^{-5} **h** 3^{-4} **i** 7^{-1} **j** 9^{-2}

k 2^{-6} **l** 3^{-2} **m** 4^{0} **n** 6^{-2} **o** 5^{-3}

p 10^{-5} **q** 2^{-7} **r** 2^{0} **s** 8^{-2} **t** 4^{-3}

2 Evaluate:

a 2^0 **b** $\left(\frac{1}{2}\right)^{-4}$ **c** $\left(\frac{2}{3}\right)^{-1}$ **d** $\left(\frac{5}{6}\right)^{-2}$ **e** $\left(\frac{x+2y}{3x-y}\right)^{0}$

f $\left(\frac{1}{5}\right)^{-3}$ **g** $\left(\frac{3}{4}\right)^{-1}$ **h** $\left(\frac{1}{7}\right)^{-2}$ **i** $\left(\frac{2}{3}\right)^{-3}$ **j** $\left(\frac{1}{2}\right)^{-5}$

k $\left(\frac{3}{7}\right)^{-1}$ **l** $\left(\frac{8}{9}\right)^{0}$ **m** $\left(\frac{6}{7}\right)^{-2}$ **n** $\left(\frac{9}{10}\right)^{-2}$ **o** $\left(\frac{6}{11}\right)^{0}$

p $\left(-\frac{1}{4}\right)^{-2}$ **q** $\left(-\frac{2}{5}\right)^{-3}$ **r** $\left(-3\frac{2}{7}\right)^{-1}$ **s** $\left(-\frac{3}{8}\right)^{0}$ **t** $\left(-1\frac{1}{4}\right)^{-2}$

3 Change into index form:

a $\frac{1}{m^3}$ **b** $\frac{1}{x}$ **c** $\frac{1}{p^7}$ **d** $\frac{1}{d^9}$ **e** $\frac{1}{k^5}$

f $\frac{1}{x^2}$ **g** $\frac{2}{x^4}$ **h** $\frac{3}{y^2}$ **i** $\frac{1}{2z^6}$ **j** $\frac{3}{5t^8}$

k $\frac{2}{7x}$ **l** $\frac{5}{2m^6}$ **m** $\frac{2}{3y^7}$ **n** $\frac{1}{(3x+4)^2}$ **o** $\frac{1}{(a+b)^8}$

p $\frac{1}{x-2}$ **q** $\frac{1}{(5p+1)^3}$ **r** $\frac{2}{(4t-9)^5}$ **s** $\frac{1}{4(x+1)^{11}}$ **t** $\frac{5}{9(a+3b)^7}$

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4 Write without negative indices:

a	t^{-5}	**b**	x^{-6}	**c**	y^{-3}	**d**	n^{-8}	**e**	w^{-10}
f	$2x^{-1}$	**g**	$3m^{-4}$	**h**	$5x^{-7}$	**i**	$(2x)^{-3}$	**j**	$(4n)^{-1}$
k	$(x+1)^{-6}$	**l**	$(8y+z)^{-1}$	**m**	$(k-3)^{-2}$	**n**	$(3x+2y)^{-9}$	**o**	$\left(\frac{1}{x}\right)^{-5}$
p	$\left(\frac{1}{y}\right)^{-10}$	**q**	$\left(\frac{2}{p}\right)^{-1}$	**r**	$\left(\frac{1}{a+b}\right)^{-2}$	**s**	$\left(\frac{x+y}{x-y}\right)^{-1}$	**t**	$\left(\frac{2w-z}{3x+y}\right)^{-7}$

1.03 Fractional indices

INVESTIGATION

FRACTIONAL INDICES

Consider the following examples.

$\left(x^{\frac{1}{2}}\right)^2 = x^1$ (by index laws)

$= x$

$\left(\sqrt{x}\right)^2 = x$

So $\left(x^{\frac{1}{2}}\right)^2 = \left(\sqrt{x}\right)^2$

$= x$

$\therefore \quad x^{\frac{1}{2}} = \sqrt{x}$

Now simplify these expressions.

1	$\left(x^2\right)^{\frac{1}{2}}$	**2**	$\sqrt{x^2}$	**3**	$\left(x^{\frac{1}{3}}\right)^3$	**4**	$\left(x^3\right)^{\frac{1}{3}}$	**5**	$\left(\sqrt[3]{x}\right)^3$
6	$\sqrt[3]{x^3}$	**7**	$\left(x^{\frac{1}{4}}\right)^4$	**8**	$\left(x^4\right)^{\frac{1}{4}}$	**9**	$\left(\sqrt[4]{x}\right)^4$	**10**	$\sqrt[4]{x^4}$

Use your results to complete:

$x^{\frac{1}{n}} =$

WS Indices

WS Fractional indices and radicals

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